Five Year Integrated M.Sc. Examination, 2023-2024
Semester - V
Code: PH-3-5-5

Time: Three Hours | Subject: Statistical Mechanics | Full Marks: 40

Questions are of values as indicated in the margin
Answer question number **one** and any **three** from the rest

1. Answer **any five** questions

$5 \times 2 = 10$

(a) What do you mean by ensemble? Define canonical ensemble.

(b) Assuming Boltzman's formula for entropy $S(E) = k_{\mathbf{B}} \ln \Omega(E)$, show that entropy is an extensive variable.

(c) "Quantum identical particles are indistinguishable"- Justify .

(d) Draw the phase space diagram of a particle falling under earth's gravitational force.

(e) Show that the canonical partition function for a non-interacting N particles system can be written as a N-th power of single particle partition function.

(f) How does the number of accessible state Ω change in case of a reversible and an irreversible process?

(g) State and explain the postulate of equal a priori probability.

2. (a) A isolated system has two non-degenerate energy states E_1 and E_2, with populations n_1 and n_2, respectively $(n_1, n_2 >> 1)$. The systems is in contact with a heat bath at temperature T. Calculate the average energy and entropy of the system.

(b) A system has three non-degenerate energy states given by $\epsilon_1/k_{\mathrm{B}} = 0\,K, \epsilon_2/k_{\mathrm{B}} = 200\,K$ and $\epsilon_3/k_{\mathrm{B}} = 300\,K$. Find the average energy $\langle E \rangle$ and dispersion $\sigma = \sqrt{\langle E^2 \rangle - \langle E \rangle^2}$ for the system if the temperature of the system is $250\,K$.

(c) Show that canonical entropy exactly matches with microcanonical entropy at thermodynamic limit (i.e.$N \to \infty$)

4+4+2=10

3. (a) Estimate the canonical entropy of an ideal gas having N particles and confined within a volume V in 3 dimension.

   (b) Discuss Gibbs paradox with appropriate example. How can one overcome this paradoxical situation by applying proper counting of states?

5+5=10

4. (a) A gas of molecules, each of mass m, is in thermal equilibrium at the absolute temperature T. Denote the velocity of a molecule by $\mathbf{v}$, its three Cartesian components by v_x, v_y, and v_z, and its speed by v. Calculate the mean value of the following quantities: (i) $\bar{v}_x$, and (ii) $\overline{v^2}_x + \overline{v^2}_y + \overline{v^2}_z$.

   (b) Show that the mean free path (l) of a molecule can be expressed as, $l = 1/(\sigma n)$, where n is the number of molecules per unit volume, and $\sigma = \pi(2R)^2$ where R is the radius of the gas molecule.

   (c) Three identical bosons can be distributed among two states. Show all possible configuration of the microstates.

(2+3)+3+2=10

5. (a) State the three quantities that are fixed in microcanonical ensemble.

   (b) Consider a microcanonical ensemble of N number of fermions with total energy E. These fermions are distributed over large number of degenerate energy levels where i-th energy level having energy ϵ_i and degeneracy g_i contains n_i number of particle. Find a expression for the number of states for a particular configuration $\{n_i\}$.

   (c) Consider an ensemble of classical one-dimensional harmonic oscillators whose energy are lying in the small range E and $E + \delta E$. Calculate the probability $P(x)dx$ that x lies between x and $x + dx$ by taking the ratio of that volume of phase space lying in the energy range *and* in the range between x and $x + dx$ to the total volume of phase space lying in the energy range between E and $E + \delta E$. Express $P(x)$ in terms of E and x.

2+4+4=10